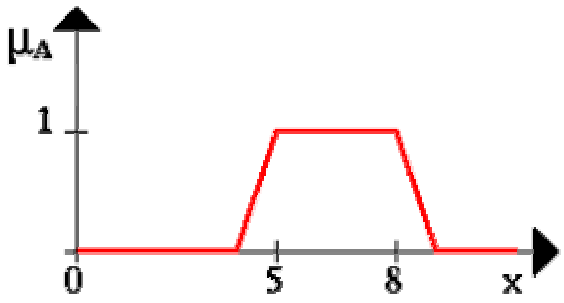
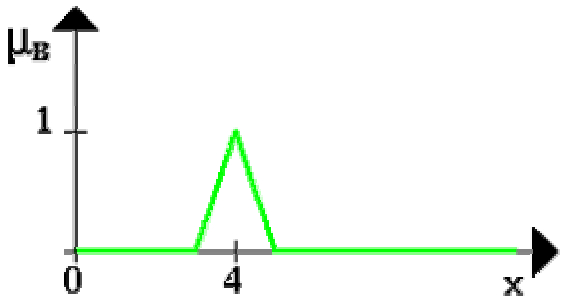
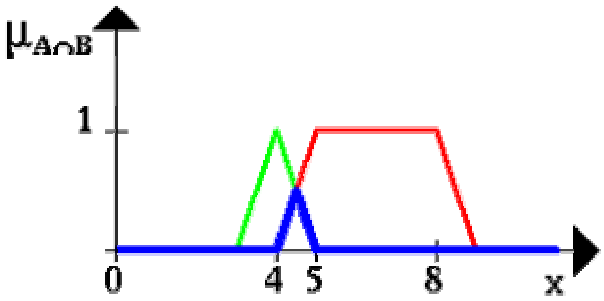
Operations on Fuzzy Sets (Logical)

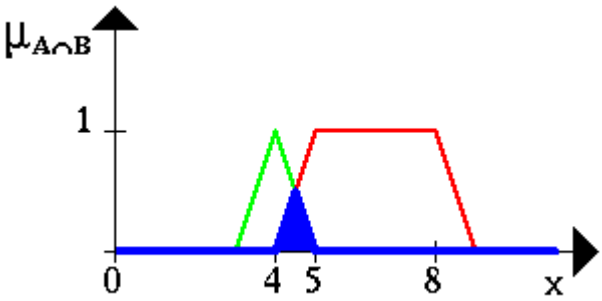
Let *A* be a fuzzy interval *between* 5 *and* 8 and *B* be a fuzzy number *about* 4. The corresponding figures are shown below.



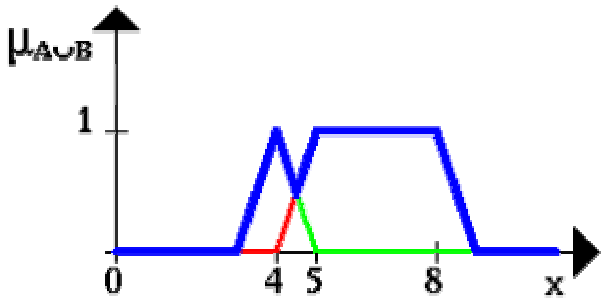


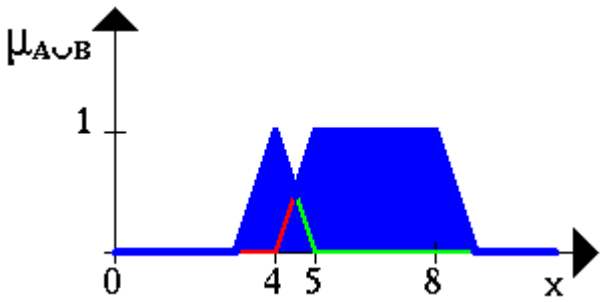
The following figure shows the fuzzy set *between* 5 *and* 8 **AND** *about* 4 (notice the blue line).



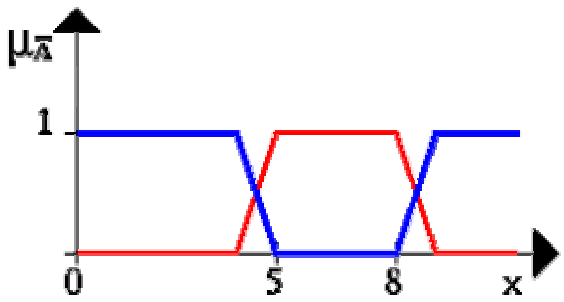


The Fuzzy set *between* 5 *and* 8 **OR** *about* 4 is shown in the next figure (again, it is the blue line).





This figure gives an example for a negation. The blue line is the **NEGATION** of the fuzzy set *A*.

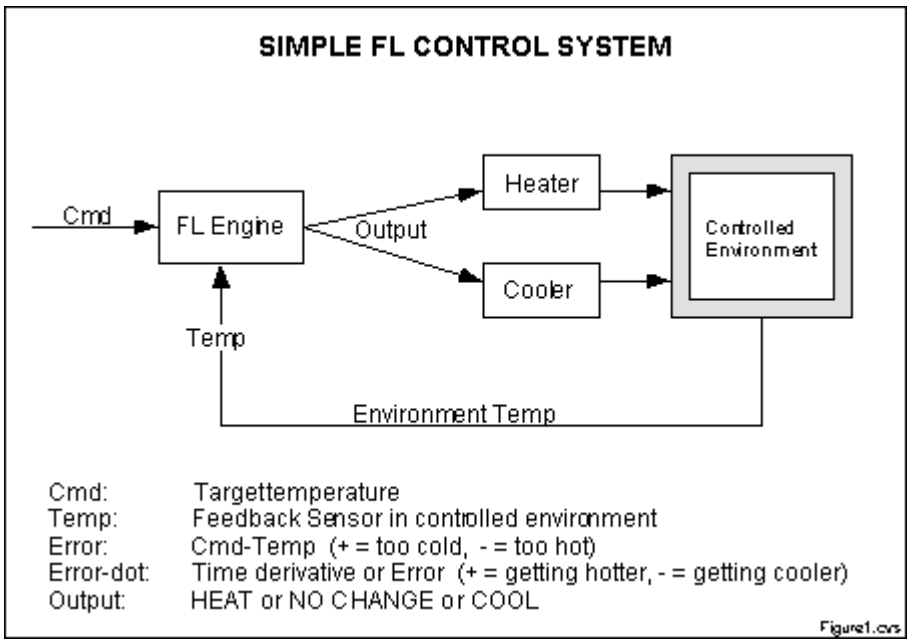


CONTROLLING A PROCESS WITH FL

The first step in implementing FL is to decide exactly what is to be controlled and how. For example:

Design a simple proportional temperature controller:

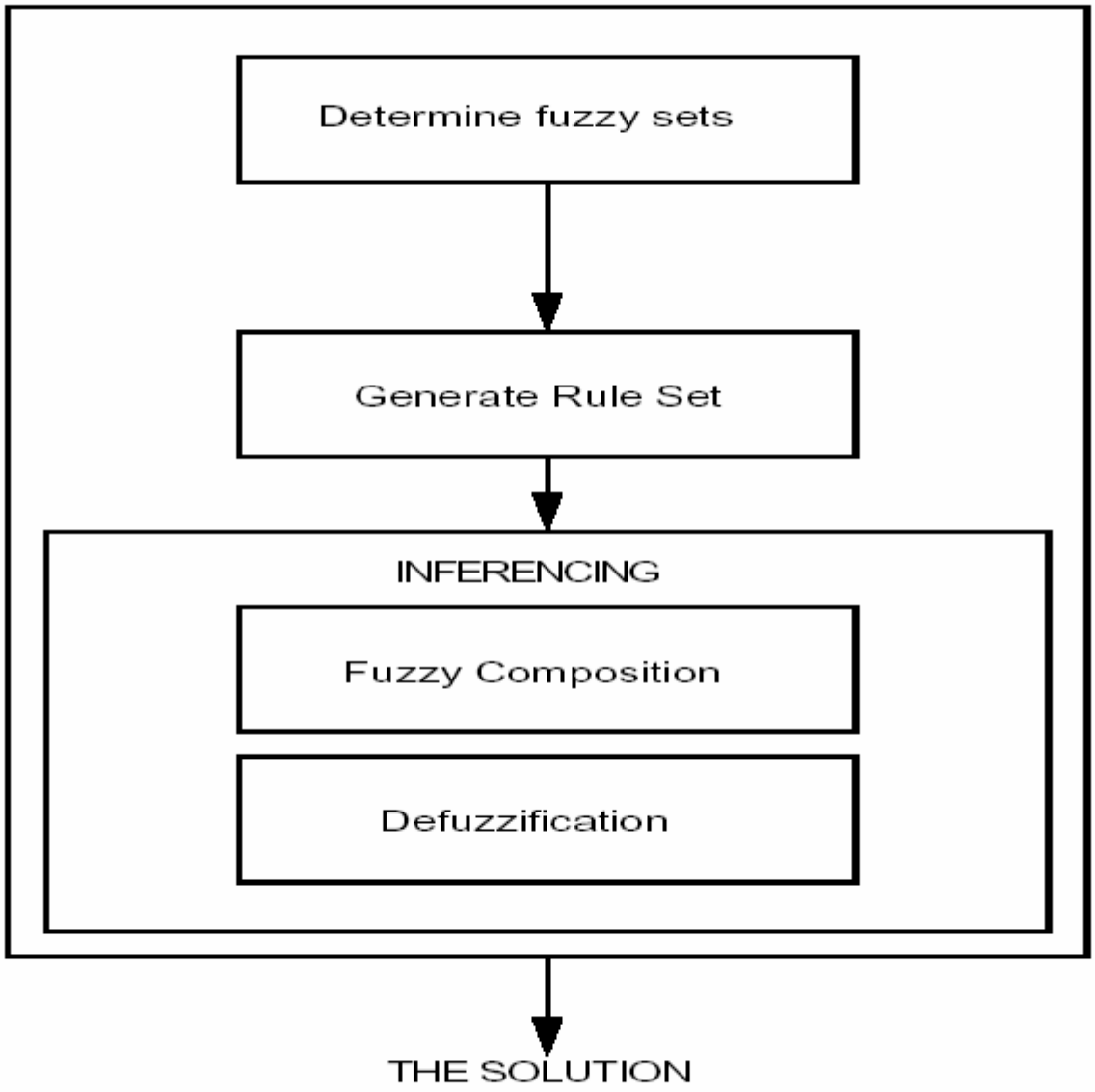
- An electric heating element
- A variable-speed cooling fan.
- A positive signal output calls for 0-100 percent heat while a negative signal output calls for 0-100 percent cooling.
- Control is achieved through proper balance and control of these two active devices.



Define the minimum number of possible input product combinations and corresponding output response conclusions using these terms. For a three-by-three matrix with heating and cooling output responses, all nine rules will need to be defined. The conclusions to the rules with the linguistic variables associated with the output response for each rule are transferred to the matrix.

Fuzzy Systems

Structure

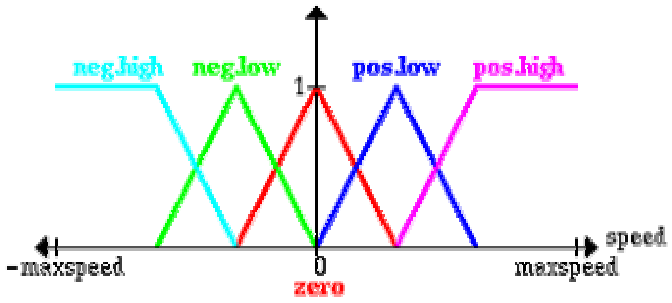


Fuzzy Control Example: Inverted pendulum

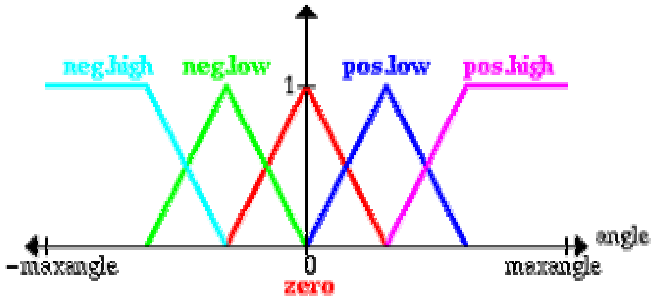
1st – Define Fuzzy Sets

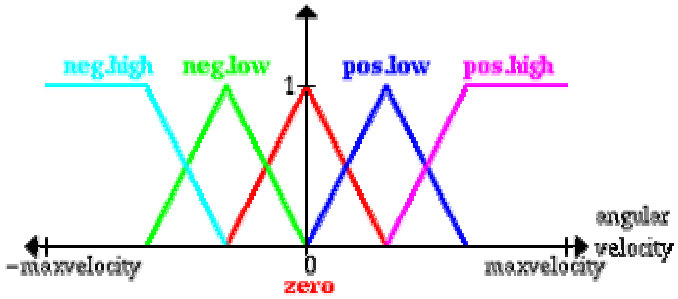
The problem is to balance a pole on a mobile platform that can move in only two directions, left or right. First of all, we have to define (subjectively) what *high* speed, *low* speed etc. of the platform is; this is done by specifying the membership functions for the fuzzy_sets

- negative high (cyan)
- negative low (green)
- zero (red)
- positive low (blue)
- positive high (magenta)



The same is done for the angle between the platform and the pendulum and the angular velocity of this angle:





Please notice that, to make it easier, we assume that in the beginning the pole is in a *nearly upright* position so that an angle greater than, say, 45 degrees in any direction can - by definition - never occur.

Fuzzy Control

2nd – Define Rules

Now we give several *rules* that say what to do in certain situations:

Consider for example that the pole is in the upright position (angle is zero) and it does not move (angular velocity is zero). Obviously this is the desired situation, and therefore we don't have to do anything (speed is zero).
Let's consider another case: the pole is in upright position as before but is in motion at *low* velocity in *positive* direction. Naturally we would have to compensate the pole's movement by moving the platform in the same direction at *low* speed.

So far we've made up two rules that can be put into a more formalized form like this:

- **If** angle is zero **and** angular velocity is zero **then** speed shall be zero.
- **If** angle is zero **and** angular velocity is pos. low **then** speed shall be pos. low.

We can summarize all applicable rules in a table:

		angle				
	speed	NH	NL	Z	PL	PH
v	NH			NH		
e	NL			NL	Z	
l	Z	NH	NL	Z	PL	PH
o	PL		Z	PL		
c	PH			PH		

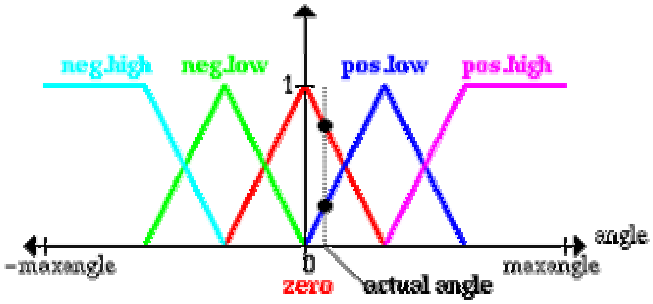
where NH is a (usual) abbreviation for negative high, NL for negative low etc.

Fuzzy Control

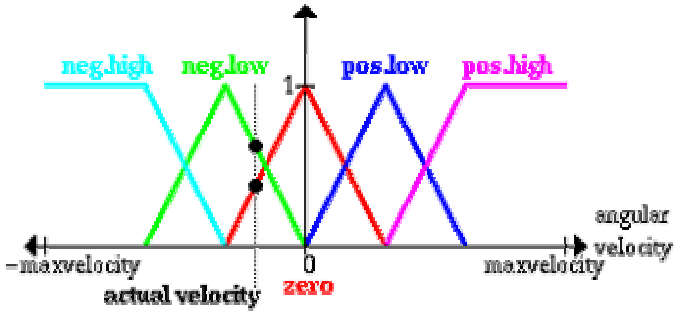
3rd – Inferencing

We are going to define two explicit values for *angle* and *angular velocity* to calculate with. Consider the following situation:

An actual value for *angle:*



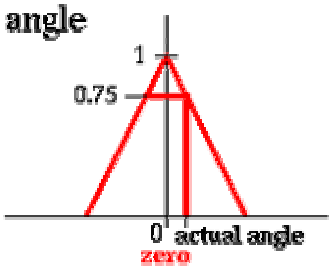
An actual value for *angular velocity:*



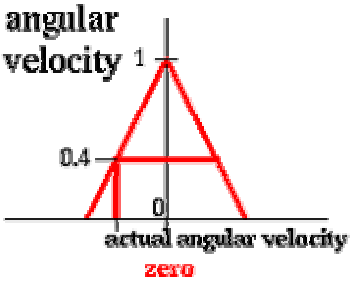
Fuzzy Control

3rd – Inferencing (cont…)

Let's apply the rule
`if angle is zero and angular velocity is zero then speed is zero`
to the values that we've selected

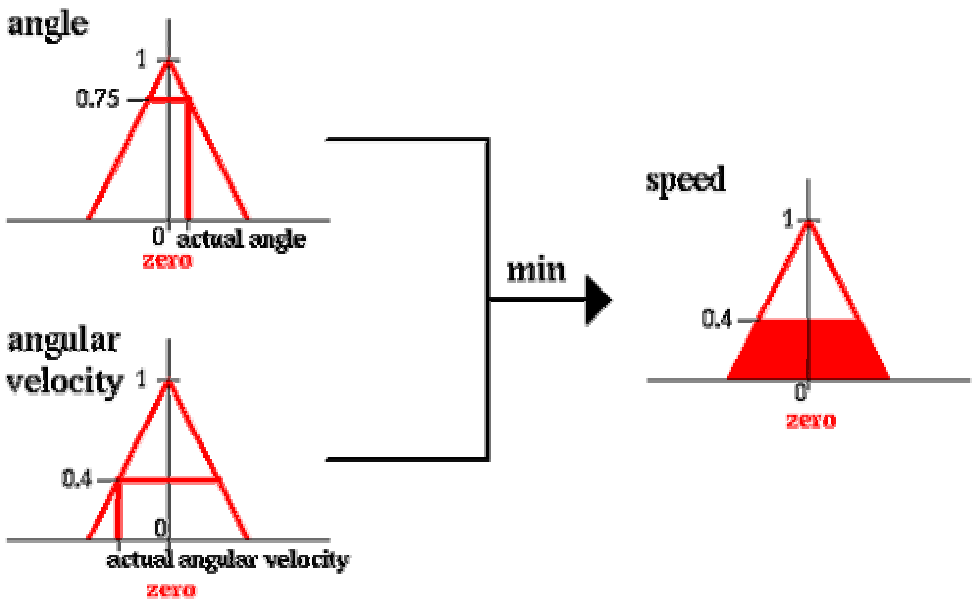


We realize that our actual value belongs to the fuzzy set "zero" to a degree of 0.75.



We realize that our actual value belongs to the fuzzy set "zero" to a degree of 0.4.

3rd – Inferencing (cont…)



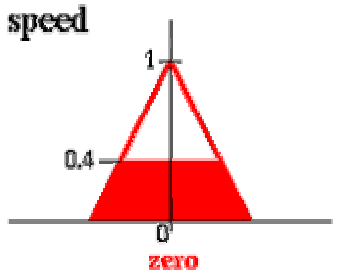
Since the two parts of the condition of our rule are connected by an AND we calculate min(0.75,0.4) = 0.4 and cut the fuzzy set "zero" of the variable "speed" at this level (according to our rule).

Fuzzy Control

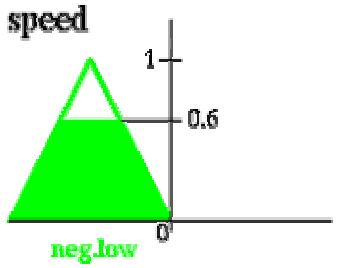
3rd – Inferencing (cont…)

Only four rules yield a result (they *fire*), and we overlap them into one single result.

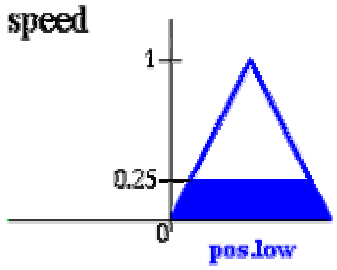
```
if angle is zero and angular velocity is zero then speed is zero
```



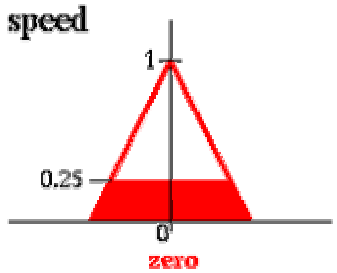

```
if angle is zero and angular velocity is negative low then speed is negative low
```




```
if angle is positive low and angular velocity is zero then speed is positive low
```



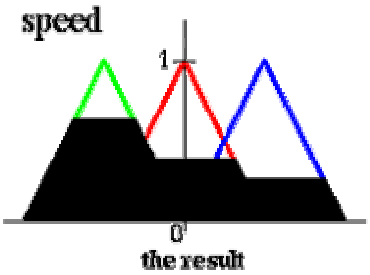

```
if angle is positive low and angular velocity is negative low then speed is zero
```



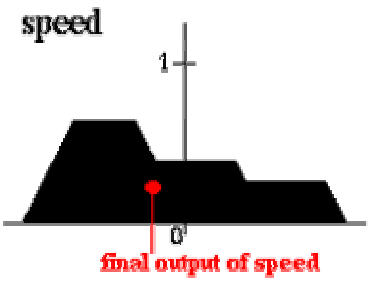
Fuzzy Control

4th – Defuzzification

These four results overlapped yield the overall result:



The result of the fuzzy controller so far is a fuzzy set (of speed), so we have to choose one representative value as the final output. There are several heuristic methods (*defuzzification methods*), one of them is e.g. to take the centre of gravity of the fuzzy set:



The whole procedure is called *Mamdani* controller.